Period 4, Oct 22, 2024

Find the domain of the function. $x \neq -3$ $\quad x \neq 7$

$$f(x)=\frac{1}{\frac{20}{x+3}-2\frac{(x+3)}{(x+3)}} = \frac{1}{\frac{20}{x+3}-\frac{2(x+3)}{x+3}} = \frac{1}{\frac{20}{x+3}-\frac{2x+6}{x+3}} = \frac{1}{\frac{20-2x-6}{x+3}} = \frac{1}{\frac{14-2x}{x+3}}$$

$$\frac{\frac{1}{1}}{\frac{2(7-x)}{x+3}} = \frac{1}{1}\cdot\frac{x+3}{2(7-x)} = \frac{x+3}{2(7-x)}$$

Domain

$(-\infty,-3)\cup(-3,7)\cup(7,\infty)$

Find the domain of the function.

$$f(x)=\frac{2x+10}{x^3+4x^2-x-4} = \frac{2x+10}{x^2(x+4)-1(x+4)} = \frac{2x+10}{(x^2-1)(x+4)} = \frac{2x+10}{(x+1)(x-1)(x+4)}$$

$x \neq -1, 1, -4$

Domain

$(-\infty,-4)\cup(-4,-1)\cup(-1,1)\cup(1,\infty)$

Find the domain of the function.

$f(x)=\sqrt{x-9}+\sqrt{x+7}$ ← No negative

$x \geq 9$ $\quad x \geq -7$

Example

$\sqrt{8-9}=\sqrt{-1}=i$

8 no Real #

$\sqrt{-8+7}=\sqrt{-1}=i$

Domain $x \geq 9$

Find $f+g$, $f-g$, fg, $\frac{f}{g}$. Determine the domain for each function.

$f(x)=\frac{7x}{x-4}$, $g(x)=\frac{2}{x+3}$

Domain $x \neq -3, 4$

$$F+g = \frac{(x+3)7x}{(x+3)(x-4)}+\frac{2(x-4)}{(x+3)(x-4)} = \frac{7x^2+21x}{(x+3)(x-4)}+\frac{2x-8}{(x+3)(x-4)} = \frac{7x^2+23x-8}{(x+3)(x-4)}$$

$$F-g = \frac{7x^2+21x}{(x+3)(x-4)}-\frac{2x-8}{(x+3)(x-4)} = \frac{7x^2+21x-2x+8}{(x+3)(x-4)} = \frac{7x^2+19x+8}{(x+3)(x-4)}$$

Domain $x \neq -3, 4$

$Fg = \left(\frac{7x}{x-4}\right)\left(\frac{2}{x+3}\right) = \frac{14x}{(x-4)(x+3)}$ $x \neq 4, -3$

$\frac{F(x)}{g(x)} = \frac{\frac{7x}{x-4}}{\frac{2}{x+3}} = \frac{7x}{x-4} \cdot \frac{x+3}{2} = \frac{7x(x+3)}{2(x-4)} = \frac{7x^2+21x}{2x-8}$

$x \neq 4, -3$

Find f + g, f − g, fg, and $\frac{f}{g}$. Determine the domain for each function.

$f(x) = \sqrt{x}$; $g(x) = x - 2$

domain $x \geq 0$

$F+g = \sqrt{x} + x - 2$ domain $x \geq 0$

$F-g = \sqrt{x} - (x-2) = \sqrt{x} - x + 2$ domain $x \geq 0$

$F \cdot g = \sqrt{x}(x-2)$ domain $x \geq 0$

$\frac{F}{g} = \frac{\sqrt{x}}{x-2}$ domain $x \geq 0$, $x \neq 2$

$[0,2) \cup (2,\infty)$

For $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{4}{x}$, find

a. $(f \circ g)(x)$; **b.** the domain of $f \circ g$

Domain $x \neq 0, -4$

$(F \circ g)(x) = F(g(x)) = \frac{g(x)}{g(x)+1} = \frac{\frac{4}{x}}{\frac{4}{x}+1} = \frac{\frac{4}{x}}{\frac{4}{x}+\frac{x}{x}} = \frac{\frac{4}{x}}{\frac{4+x}{x}}$

$\frac{4}{x} \cdot \frac{x}{(4+x)} = \frac{4\cancel{x}}{\cancel{x}(x+4)} = \frac{4}{x+4}$

$x \neq -4$

$g(6)=6+3=9$

For $f(x)=\sqrt{x}$ and $g(x)=x+3$, find the following functions.

a. $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(6)$; **d.** $(g \circ f)(6)$

domain $x \geq -3$

$(F\circ g)(x)=F(g(x))=\sqrt{g(x)}=\sqrt{x+3}$

$(g\circ F)(x)=g(F(x))=F(x)+3=\sqrt{x}+3 \Rightarrow$ domain $x \geq 0$

$(F\circ g)(6)=F(g(6))=F(9)=\sqrt{9}=3$

$(F\circ g)(x)=\sqrt{x+3}$

$(F\circ g)(6)=\sqrt{6+3}=\sqrt{9}=3$

$(g\circ F)(x)=\sqrt{x}+3$

$(g\circ F)(6)=\sqrt{6}+3$

For $f(x)=x^2+8$ and $g(x)=x^2-1$, find the following functions.

a. $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(2)$; **d.** $(g \circ f)(2)$

Domain $\mathbb{R}$

$(F\circ g)(x)=F(g(x))=(g(x))^2+8=(x^2-1)^2+8=(x^2-1)(x^2-1)+8$

$x^4-2x^2+1+8=x^4-2x^2+9$

$(g\circ F)(x)=g(F(x))=(F(x))^2-1=(x^2+8)^2-1$

$(x^2+8)(x^2+8)-1$

$x^4+16x^2+64-1=x^4+16x^2+63$

$(F\circ g)(2)=2^4-2(2)^2+9=16-8+9=17$

$(g\circ F)(2)=(2)^4+16(2)^2+63=16+64+63=80+63=143$

Simplify the complex fraction.

$x^2-2^2=(x-2)(x+2)$

$$\frac{\frac{4}{x+2}-\frac{4}{x-2}}{\frac{13}{x^2-4}} = \frac{\frac{4(x-2)}{(x+2)(x-2)}-\frac{4(x+2)}{(x-2)(x+2)}}{\frac{13}{(x-2)(x+2)}} = \frac{\frac{4x-8}{(x+2)(x-2)}-\frac{4x+8}{(x-2)(x+2)}}{\frac{13}{(x-2)(x+2)}}$$

$$\frac{\frac{\cancel{4x}-8-\cancel{4x}-8}{(x+2)(x-2)}}{\frac{13}{(x-2)(x+2)}} = \frac{-16}{(x+2)(x-2)} \cdot \frac{(x-2)(x+2)}{13} = \frac{-16\cancel{(x-2)}\cancel{(x+2)}}{13\cancel{(x+2)}\cancel{(x-2)}} = \frac{-16}{13}$$

$x \neq 2, -2$

Subtract.

$$\frac{3x+5}{x^2-9x+18}-\frac{3}{x-3}$$

$1x^2-9x+18$

$1 \cdot 18 = 18$

$-6 \quad -3 = -9$

$x^2-6x-3x+18$

$x(x-6)-3(x-6)$

$(x-6)(x-3)$

$$\frac{3x+5}{(x-6)(x-3)} - \frac{3(x-6)}{(x-3)(x-6)}$$

$$\frac{3x+5}{(x-6)(x-3)} - \frac{3x-18}{(x-3)(x-6)}$$

$$\frac{\cancel{3x}+5-\cancel{3x}+18}{(x-6)(x-3)} = \frac{23}{(x-6)(x-3)}$$

$x \neq 6, 3$

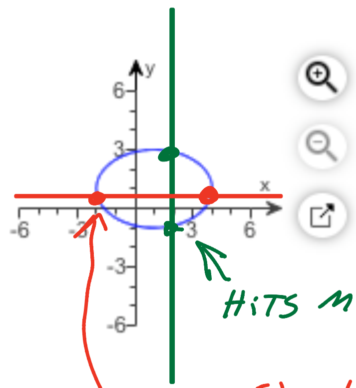
F(x) has an inverse g(x)

x	F(x)
0	5
1	7
-1	3
2	-6

x	g(x)
5	0
7	1
3	-1
-6	2

Vertical Line test

is This a Function?



Hits more Then once not a Function

Check To see if inverse is a Function

Horizontial Line Test

inverse is not a Function Horizontail Line Test Hits more Than once

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Find f(g(x)) and g(f(x)) and determine whether the pair of functions f and g are inverses of each other.

$$f(x) = \frac{8}{x-9} \quad \text{and} \quad g(x) = \frac{8}{x} + 9$$

$$(f \circ g)(x) = f(g(x)) = \frac{8}{g(x)-9}$$

$$= \frac{8}{\frac{8}{x} + \cancel{9} - \cancel{9}} = \frac{\frac{8}{1}}{\frac{8}{x}}$$

$$\frac{\cancel{8}}{1} \cdot \frac{x}{\cancel{8}} = x$$

If you get an X Then inverse

To Find The inverse of F(x)

$$x = \frac{8}{y-9} \quad \text{Solve For } y$$

$$(y-9)x = \frac{8}{\cancel{y-9}}\cancel{(y-9)}$$

$$\frac{\cancel{x}(y-9)}{\cancel{x}} = \frac{8}{x}$$

$$y - 9 = \frac{8}{x}$$

$$+9 \qquad +9$$

$$y = \frac{8}{x} + 9$$

$$g(f(x)) = \frac{8}{f(x)} + 9 = \frac{8}{\frac{8}{x-9}} + 9$$

$$\frac{\frac{8}{1}}{\frac{8}{x-9}} + 9 = \frac{\cancel{8}}{1} \cdot \frac{(x-9)}{\cancel{8}} + 9 = x \cancel{-9} \cancel{+9} = x$$

The function $f(x) = (x + 7)^3$ is one-to-one.

a. Find an equation for $f^{-1}(x)$, the inverse function.

b. Verify that your equation is correct by showing that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}(f(x)) = x$.

$F(x) = (x+7)^3$ inverse $\sqrt[3]{x} = \sqrt[3]{(y+7)^3}$

$\sqrt[3]{x} = y + 7$

$-7 \qquad -7$

$\sqrt[3]{x} - 7 = y \leftarrow F^{-1}(x) =$ inverse Function of $F(x)$

$F(F^{-1}(x)) = (F^{-1}(x) + 7)^3 = (\sqrt[3]{x} - \cancel{7} + \cancel{7})^3 = (\sqrt[3]{x})^3 = x$

$F^{-1}(F(x)) = \sqrt[3]{F(x)} - 7 = \sqrt[3]{(x+7)^3} - 7 = x + 7 - 7 = x$

The function $f(x) = \dfrac{3x + 1}{x - 2}$ is one-to-one.

a. Find an equation for $f^{-1}(x)$, the inverse function.

b. Verify that your equation is correct by showing that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}(f(x)) = x$.

$(y-2)\, x = \dfrac{3y+1}{\cancel{y-2}} \cancel{(y-2)}$

$(y-2)x = 3y + 1$

$xy - 2x = 3y + 1$

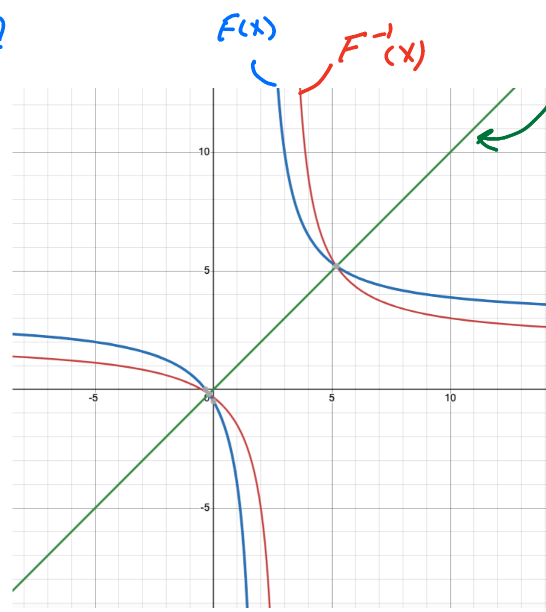
$-3y \quad +2x \qquad -3y \quad +2x$

$xy - 3y = 2x + 1$

$\dfrac{y\cancel{(x-3)}}{\cancel{(x-3)}} = \dfrac{2x+1}{(x-3)}$

$y = \dfrac{2x+1}{x-3}$

Reflected over $y = x$



Given the function $f(x) = (x-1)^2$, $x \le 1$, complete parts a through c.

(a) Find an equation for $f^{-1}(x)$.

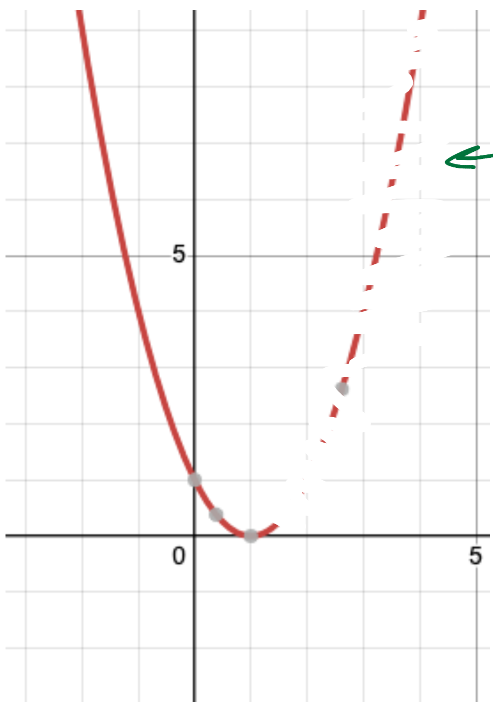
(b) Graph f and f^{-1} in the same rectangular coordinate system.

(c) Use interval notation to give the domain and the range of f and f^{-1}.

$F(x) = (x-1)^2$

x	F(x)	
1	0	$=(1-1)^2$
0	1	$=(0-1)^2$
-1	4	$=(-1-1)^2$
-2	9	$=(-2-1)^2$

(1,0)
(0,1)
(-1,4)
(-2,9)



not There

$\sqrt{x} = \sqrt{(y-1)^2}$

$\pm\sqrt{x} = y-1$

$+1 \qquad +1$

$\sqrt{x} + 1 = y$

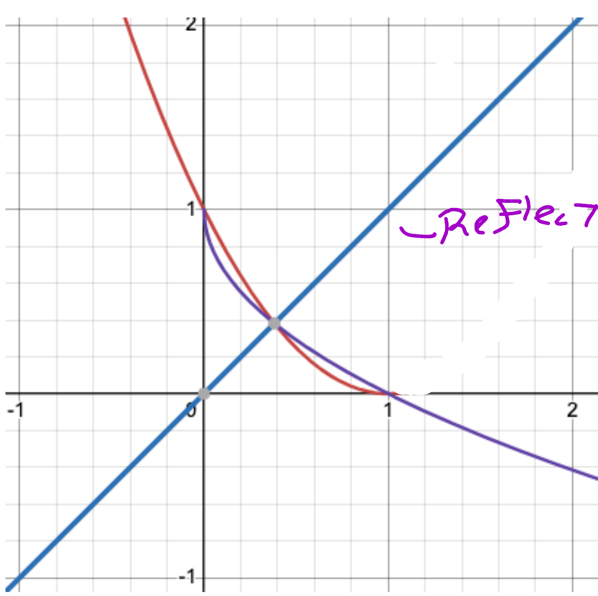
$-\sqrt{x} + 1 = y$

inverse

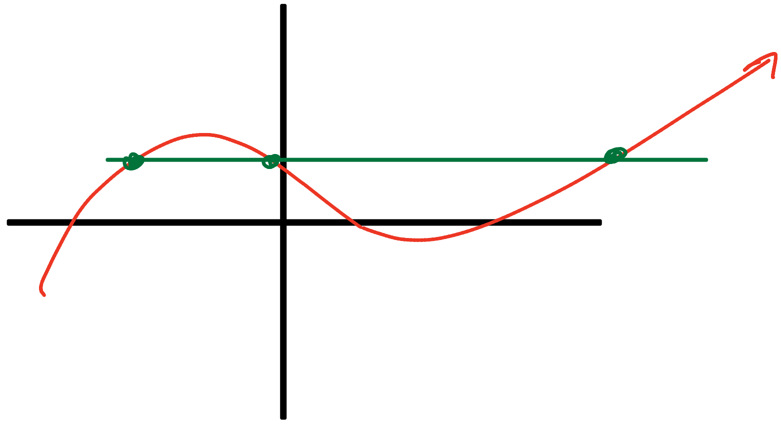
(0,1)
~~(1,2)~~
(1,0)
~~(4,3)~~
(4,-)
~~(4,4)~~
(9,-2)

inverse

x	$y=\sqrt{x}+1$	
0	1	$=\pm\sqrt{0}+1$
1	2	$=+\sqrt{1}+1$
1	0	$=-\sqrt{1}+1$
4	3	$=+\sqrt{4}+1$
4	-1	$=-\sqrt{4}+1$
9	4	$\sqrt{9}+1$
9	-2	$-\sqrt{9}+1$

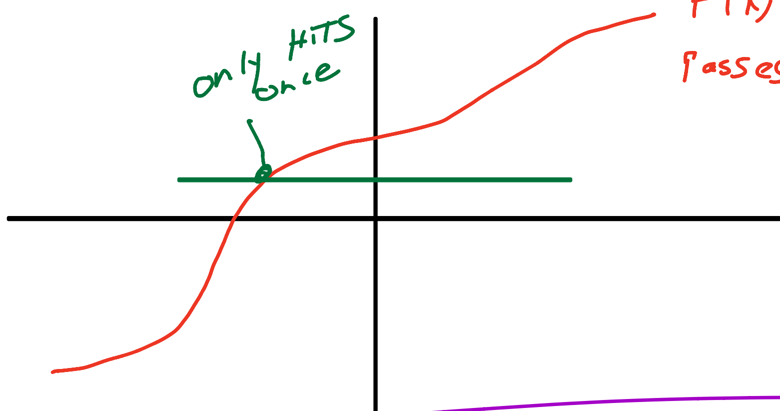


Reflected over $y = x$



does F(X) have an inverse Function

No HLT HiTS 3 Times



F(X) Has an inverse

Passes HLT

The functions f and g are defined by the following tables. Use the tables to evaluate the given composite function.

$f^{-1}(g(10)) = F^{-1}(4) \Rightarrow (4, ?) = 0$

x	f(x)
−4	2
0	4
1	5
3	−5

x	g(x)
−3	0
2	1
5	2
10	4

$F(?) = 4 \quad (?, 4)$

(−4, 2)
(0, 4)
(1, 5)
(3, −5)

$F^{-1}(x)$ because $F(x)$

(4, 0) (0, 4)